1 (i) Solve the equation $10^x = 316$. [2]

(ii) Simplify $\log_a(a^2) - 4\log_a\left(\frac{1}{a}\right)$. [3]

2 **Answer part (iii) of this question on the insert provided.**

A hot drink is made and left to cool. The table shows its temperature at ten-minute intervals after it is made.

Time (minutes)	10	20	30	40	50
Temperature (°C)	68	53	42	36	31

The room temperature is 22 °C. The difference between the temperature of the drink and room temperature at time t minutes is z °C. The relationship between z and t is modelled by

$$z = z_0 10^{-kt},$$

where z_0 and k are positive constants.

(i) Give a physical interpretation for the constant z_0. [2]

(ii) Show that $\log_{10} z = -kt + \log_{10} z_0$. [2]

(iii) On the insert, complete the table and draw the graph of $\log_{10} z$ against t.

Use your graph to estimate the values of k and z_0.

Hence estimate the temperature of the drink 70 minutes after it is made. [9]

3 **(a)** André is playing a game where he makes piles of counters. He puts 3 counters in the first pile. Each successive pile he makes has 2 more counters in it than the previous one.

(i) How many counters are there in his sixth pile? [1]

(ii) André makes ten piles of counters. How many counters has he used altogether? [2]

(b) In another game, played with an ordinary fair die and counters, Betty needs to throw a six to start.

The probability P_n of Betty starting on her nth throw is given by

$$P_n = \frac{1}{6} \times \left(\frac{5}{6}\right)^{n-1}.$$

(i) Calculate P_4. Give your answer as a fraction. [2]

(ii) The values $P_1, P_2, P_3, \ldots$ form an infinite geometric progression. State the first term and the common ratio of this progression.

Hence show that $P_1 + P_2 + P_3 + \ldots = 1$. [3]

(iii) Given that $P_n < 0.001$, show that n satisfies the inequality

$$n > \frac{\log_{10} 0.006}{\log_{10}\left(\frac{5}{6}\right)} + 1.$$

Hence find the least value of n for which $P_n < 0.001$. [4]